

Advanced Sequences

Teacher guide and model answers

Chapter 2 | Module 9 | Student Book pages 85-90 | Suggested time: 50 minutes | 20 marks



Learning goals

- Explain and apply difference sequence with valid conditions and clear reasoning.
- Explain and apply reconstruction from differences with valid conditions and clear reasoning.
- Explain and apply grouped sequence with valid conditions and clear reasoning.

Quick reference

Difference sequence	A difference sequence records $a_{n+1} - a_n$ and can reveal a simpler pattern.
Reconstruction from differences	An original term equals the initial term plus the sum of preceding differences.
Grouped sequence	A grouped sequence changes its grouping pattern, so position must be located by cumulative group sizes.

Worked example

The sequence is defined by $u_n = 3n^2 + 2n$. Find $u_{n+1} - u_n$ when $n=8$. Worked solution: Subtract before substituting: $u_{n+1} - u_n = 6n + 5$; at $n=8$, the value is 53.

Graduated practice

Answer without viewing the key. Show complete working for short-response questions.

1. If $u_n = 2n^2 - 3n$, what is $u_{(n+1)} - u_n$? [1 marks]

- A) $2n-3$ B) $4n-1$
C) $4n+1$ D) $2n^2-1$

Answer: $4n-1$

Substitute and subtract: $2(n+1)^2 - 3(n+1) - (2n^2 - 3n) = 4n - 1$.

2. If $a_1 = 4$ and $a_{(n+1)} - a_n = 3n + 1$, find a_5 . [1 marks]

- A) 34 B) 36
C) 38 D) 42

Answer: 38

$a_5 = a_1 + \sum \text{from } n=1 \text{ to } 4 \text{ of } (3n+1) = 4 + 30 + 4 = 38$.

3. In the grouped sequence 1; 2,2; 3,3,3; 4,4,4,4; ... what is term 14? [1 marks]

- A) 4 B) 5
C) 6 D) 14

Answer: 5

There are $1+2+3+4=10$ terms through group 4, so group 5 occupies positions 11 to 15.

4. If group sizes are 1,2,3,..., in which group and local position is term 23? [1 marks]

- A) Group 6, position 2 B) Group 7, position 2
C) Group 7, position 3 D) Group 8, position 1

Answer: Group 7, position 2

There are 21 terms through group 6; therefore term 23 is in group 7 at local position $23-21=2$.

5. The sequence 2,6,12,20,... is proposed to follow $a_n = n(n+1)$. Which next term supports the rule? [1 marks]

- A) 28 B) 30
C) 32 D) 40

Answer: 30

At $n=5$ the rule gives $5 \times 6 = 30$, and the differences 4,6,8,10 fit the structure.

6. Rows contain 12,16,20,... seats. Which formula models the number of seats in row n ? [1 marks]

A) $a_n = 4n + 8$

B) $a_n = 12n + 4$

C) $a_n = 4n + 12$

D) $a_n = 8n + 4$

Answer: $a_n = 4n + 8$

The first term is 12 and the difference is 4, so $a_n = 12 + 4(n-1) = 4n + 8$.

7. True or false: constant second differences suggest that a quadratic model may fit a sequence. [1 marks]

☐ True ☐ False

Answer: True

A quadratic sequence has arithmetic first differences and therefore constant second differences.

8. True or false: knowing only the difference sequence uniquely determines the original sequence. [1 marks]

☐ True ☐ False

Answer: False

An initial term is also required; sequences with the same differences may differ by a constant.

9. In the pattern 1,1; 2,2,2; 3,3,3,3; ... group g contains $g+1$ copies of g . Find term 20 and justify. [3 marks]

Answer: 5

The sizes 2,3,4,5,6 total 20 through group 5, so term 20 is the last 5 in group 5.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

10. Group sizes are 2,4,6,8,... Identify the group and local position of term 35. [3 marks]

Answer: Group 6, position 5

The first five groups contain $2+4+6+8+10=30$ terms. Group 6 has size 12, so the local position is $35-30=5$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

11. For 3,8,15,24,... construct a quadratic general term, then use it to find a_6 . [3 marks]

Answer: $a_n = n^2 + 2n$ and $a_6 = 48$

The differences 5,7,9 have constant second difference 2. The formula $n^2 + 2n$ reproduces the data and gives $a_6 = 36 + 12 = 48$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

12. A tank receives 5,8,11,... litres in successive minutes. Model the amount in minute n and find the total after 12 minutes. [3 marks]

Answer: $a_n = 3n + 2$ and the total is 258 litres

The difference is 3, so $a_n = 5 + 3(n-1) = 3n + 2$. Then $S_{12} = 12[2 \times 5 + 11 \times 3]/2 = 258$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

Challenge and review

1. Given a_1 , first difference d_1 and constant second difference c , derive a formula for a_n .

The first differences are $d_1, d_1+c, d_1+2c, \dots$, so $a_n = a_1 + (n-1)d_1 + c(n-1)(n-2)/2$.

2. In the sequence 1; 2,2; 3,3,3; ... find term 2026 and its local position within its group.

The first 63 groups contain $63 \times 64 / 2 = 2016$ terms and the first 64 contain 2080, so term 2026 is 64 at local position 10.

3. Error analysis: a student fits a rule to the first three terms and declares it proved for every term. Explain why further structural validation is required.

Matching finitely many terms does not exclude other rules that agree initially and later differ; the difference, grouping rule or assumed model class must also be checked.

Follow-up plan

Below 14/20: revisit summing differences, cumulative group sizes, and local position, then retry Questions 2-6 and 9-12. At 14 or above: continue to the online module challenge.