# Advanced Sequences

Printable teaching and practice sheet

Chapter 2 | Module 9 | Student Book pages 85-90 | Suggested time: 50 minutes | 20 marks

Name: _____ Class: _____ Date: _____

## Learning goals

- Explain and apply difference sequence with valid conditions and clear reasoning.
- Explain and apply reconstruction from differences with valid conditions and clear reasoning.
- Explain and apply grouped sequence with valid conditions and clear reasoning.

## Quick reference

| | |
|---|---|
| **Difference sequence** | A difference sequence records a_{n+1}-a_n and can reveal a simpler pattern. |
| **Reconstruction from differences** | An original term equals the initial term plus the sum of preceding differences. |
| **Grouped sequence** | A grouped sequence changes its grouping pattern, so position must be located by cumulative group sizes. |

## Worked example

The sequence is defined by u_n=3n^2+2n. Find u_(n+1)-u_n when n=8. Worked solution: Subtract before substituting: u_(n+1)-u_n=6n+5; at n=8, the value is 53.

# Graduated practice

Answer without viewing the key. Show complete working for short-response questions.

**1. If u_n=2n^2-3n, what is u_(n+1)-u_n? [1 marks]**

A) 2n-3
B) 4n-1
C) 4n+1
D) 2n^2-1

**2. If a1=4 and a_(n+1)-a_n=3n+1, find a5. [1 marks]**

A) 34
B) 36
C) 38
D) 42

**3. In the grouped sequence 1; 2,2; 3,3,3; 4,4,4,4; ... what is term 14? [1 marks]**

A) 4
B) 5
C) 6
D) 14

**4. If group sizes are 1,2,3,..., in which group and local position is term 23? [1 marks]**

A) Group 6, position 2
B) Group 7, position 2
C) Group 7, position 3
D) Group 8, position 1

**5. The sequence 2,6,12,20,... is proposed to follow a_n=n(n+1). Which next term supports the rule? [1 marks]**

A) 28
B) 30
C) 32
D) 40

**6. Rows contain 12,16,20,... seats. Which formula models the number of seats in row n? [1 marks]**

A) a_n=4n+8
B) a_n=12n+4
C) a_n=4n+12
D) a_n=8n+4

**7. True or false: constant second differences suggest that a quadratic model may fit a sequence. [1 marks]**

□ True □ False

**8. True or false: knowing only the difference sequence uniquely determines the original sequence. [1 marks]**

□ True □ False

**9. In the pattern 1,1; 2,2,2; 3,3,3,3; ... group g contains g+1 copies of g. Find term 20 and justify. [3 marks]**

**10. Group sizes are 2,4,6,8,... Identify the group and local position of term 35. [3 marks]**

**11. For 3,8,15,24,... construct a quadratic general term, then use it to find a6. [3 marks]**

**12. A tank receives 5,8,11,... litres in successive minutes. Model the amount in minute n and find the total after 12 minutes. [3 marks]**

# Challenge and review

**1. Given a1, first difference d1 and constant second difference c, derive a formula for a_n.**

**2. In the sequence 1; 2,2; 3,3,3; ... find term 2026 and its local position within its group.**

**3. Error analysis: a student fits a rule to the first three terms and declares it proved for every term. Explain why further structural validation is required.**

## Follow-up plan

Below 14/20: revisit summing differences, cumulative group sizes, and local position, then retry Questions 2-6 and 9-12. At 14 or above: continue to the online module challenge.