Summation

Teacher guide and model answers

Chapter 2 | Module 8 | Student Book pages 68-84 | Suggested time: 50 minutes | 20 marks

Learning goals

- Explain and apply sigma notation with valid conditions and clear reasoning.
- Explain and apply summation laws with valid conditions and clear reasoning.
- Explain and apply standard sums with valid conditions and clear reasoning.

Quick reference

Sigma notation	Sigma notation records an indexed sum with an expression, lower limit, and upper limit.
Summation laws	Sums distribute over addition and constant factors can be taken outside.
Standard sums	Known formulas for Σk, Σk^2, and geometric sums turn patterns into closed forms.

Worked example

Evaluate $\Sigma(k=1 \text{ to } 8)\ (6k+4)$. Worked solution: Use linearity: $6\Sigma k+(4)\Sigma 1 = 6\cdot 8\cdot 9/2 + (4)\cdot 8 = 248$.

Graduated practice

Answer without viewing the key. Show complete working for short-response questions.

1. Which expanded expression equals Σ from k=2 to 5 of (2k-1)? [1 marks]

A) 1+3+5+7
B) 3+5+7+9
C) 3+5+7
D) 2+4+6+8

Answer: 3+5+7+9

Substitute k=2, 3, 4 and 5 into 2k-1.

2. Evaluate Σ from k=1 to 6 of (3k+2) using summation laws. [1 marks]

A) 63
B) 69
C) 75
D) 81

Answer: 75

3Σk+2Σ1=3(6×7/2)+2×6=63+12=75.

3. Evaluate Σ from k=1 to 5 of k^2. [1 marks]

A) 25
B) 30
C) 55
D) 225

Answer: 55

Use Σk^2=n(n+1)(2n+1)/6: 5×6×11/6=55.

4. Evaluate Σ from k=1 to 8 of 1/[k(k+1)]. [1 marks]

A) 1/9
B) 7/8
C) 8/9
D) 9/8

Answer: 8/9

Since 1/[k(k+1)]=1/k-1/(k+1), intermediate terms cancel and 1-1/9=8/9 remains.

5. Evaluate Σ from k=1 to 4 of k/2^k. [1 marks]

A) 11/8
B) 3/2
C) 13/8
D) 15/8

Answer: 13/8

The sum is 1/2+2/4+3/8+4/16=13/8.

6. If S_n=2n^2+3n, what is a5? [1 marks]

A) 17 B) 19

C) 21 D) 65

Answer: 21

a5=S5-S4=65-44=21.

7. True or false: the number of integer indices from lower limit p to upper limit q is q-p+1. [1 marks]

□ True □ False

Answer: True

The endpoints are both included, so 1 is added.

8. True or false: Σ from k=1 to n of the constant c equals c. [1 marks]

□ True □ False

Answer: False

The constant c occurs n times, so the sum is nc.

9. Evaluate Σ from k=1 to 6 of (2k^2-3k+1) using standard sums. [3 marks]

Answer: 125

2Σk^2-3Σk+Σ1=2×91-3×21+6=125.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

10. Evaluate Σ from k=1 to 20 of [1/k-1/(k+1)], showing which terms cancel. [3 marks]

Answer: 20/21

The -1/2 cancels +1/2 and so on, leaving 1-1/21=20/21.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

11. Evaluate Σ from k=1 to 5 of k×2^(k-1), displaying the terms before adding. [3 marks]

Answer: 129

The sum is 1+4+12+32+80=129.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

12. If $S_n=n(n+2)$, find a formula for a_n and then calculate a_{10}. State what happens at $n=1$. [3 marks]

Answer: $a_n=2n+1$ and $a_{10}=21$

$a_1=S_1=3$. For $n\geq 2$, $a_n=S_n-S_{n-1}=n(n+2)-(n-1)(n+1)=2n+1$, which also holds at $n=1$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

Challenge and review

1. Decompose 1/[(k+2)(k+3)] into partial fractions, then find its sum from k=1 to n in closed form.

1/[(k+2)(k+3)]=1/(k+2)-1/(k+3), so telescoping leaves 1/3-1/(n+3).

2. In round k, a competition awards EGP 50k(0.8)^(k-1). Find the total award over the first four rounds.

The total is 50+80+96+102.4=EGP 328.4.

3. Error analysis: a student says the number of terms in a sum from k=4 to n is n-4. Explain and correct the count.

Both endpoints are included, so the correct count is n-4+1=n-3 terms.

Follow-up plan

Below 14/20: revisit term counting, sigma linearity, telescoping and recovering a_n from S_n, then retry Questions 3-6 and 9-12. At 14 or above: continue to the online module challenge.