

Geometric Sequences

Teacher guide and model answers

Chapter 2 | Module 7 | Student Book pages 57-67 | Suggested time: 50 minutes | 20 marks



Learning goals

- Explain and apply common ratio with valid conditions and clear reasoning.
- Explain and apply general term with valid conditions and clear reasoning.
- Explain and apply finding parameters with valid conditions and clear reasoning.

Quick reference

Common ratio	A geometric sequence has constant ratio $r = \frac{a_{n+1}}{a_n}$ for nonzero terms.
General term	A geometric sequence has $a_n = a_1 r^{(n-1)}$.
Finding parameters	Two suitable term conditions can determine a_1 and r , sometimes with more than one real possibility.

Worked example

A geometric sequence has $a_1=1$ and ratio 2. Find a_6 . Worked solution: $a_6 = a_1 \cdot r^{(6-1)} = 1 \cdot 2^5 = 32$.

Graduated practice

Answer without viewing the key. Show complete working for short-response questions.

1. For the geometric sequence 162, 54, 18, 6, ... what is the common ratio? [1 marks]

- A) 3
B) $\frac{1}{3}$
C) -3
D) -108

Answer: $\frac{1}{3}$

Divide a term by its predecessor: $54/162=18/54=1/3$.

2. A geometric sequence has first term 5 and common ratio 2. Find a_7 . [1 marks]

- A) 160
B) 315
C) 320
D) 640

Answer: 320

$a_7=5 \times 2^{(7-1)}=5 \times 64=320$.

3. In a positive geometric sequence, $a_3=12$ and $a_6=96$. Find a_1 and r . [1 marks]

- A) $a_1=3, r=2$
B) $a_1=2, r=3$
C) $a_1=6, r=2$
D) $a_1=3, r=8$

Answer: $a_1=3, r=2$

Divide the conditions: $r^3=96/12=8$, so $r=2$; then $a_1=12/2^2=3$.

4. A geometric sequence has first term 3 and common ratio 2. Find the sum of its first 6 terms. [1 marks]

- A) 96
B) 126
C) 189
D) 192

Answer: 189

$S_6=3(2^6-1)/(2-1)=3 \times 63=189$.

5. If $a_1=81$ and $r=-1/3$, what is a_5 ? [1 marks]

- A) -3
B) -1
C) 1
D) 3

Answer: 1

$a_5=81(-1/3)^4=81 \times 1/81=1$; the even power gives a positive sign.

A) 310
B) 400
C) 432
D) 450

The growth factor is 1.20, so the count is $250(1.2)^3=432$.

☐ True ☐ False

This is the defining property of a geometric sequence.

☐ True ☐ False

A negative ratio makes the signs alternate; it does not make every term negative.

$$r^3 = 162/6 = 27, \text{ so } r=3; \text{ then } a_1 = 6/3 = 2.$$

Here $a_1=2$ and $r=3$, so $S_7=2(3^7-1)/(3-1)=2186$.

$a_7=64(1/2)^6=1$ and $S_7=64[1-(1/2)^7]/(1-1/2)=127$; term magnitudes decrease.

3

12. A model population is 800 and decreases by 10% each year. Find the population after four years of decrease and state the change factor. [3 marks]

Answer: 524.88, with change factor 0.9

A 10% decrease means multiplying by 0.9 each year: $800(0.9)^4=524.88$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

Challenge and review

1. Derive the finite geometric-sum formula for $r \neq 1$ by writing S_n and rS_n , then subtracting.

From $S_n = a + ar + \dots + ar^{n-1}$ and $rS_n = ar + \dots + ar^n$, subtraction gives $(r-1)S_n = a(r^n - 1)$, so $S_n = a(r^n - 1)/(r-1)$.

2. A device worth EGP 20,000 loses 12% of its value each year. Write a model after n years and find its value after 5 years to the nearest pound.

The retention factor is 0.88, so $V_n = 20000(0.88)^n$ and $V_5 \approx \text{EGP } 10,555$.

3. Error analysis: a student says 3, 6, 12, 24 is arithmetic because the increases grow regularly. Explain the correct classification.

The differences 3, 6, 12 are not constant, but the ratios are all 2; therefore the sequence is geometric with ratio 2.

Follow-up plan

Below 14/20: revisit ratio versus difference and the formulas for a_n and S_n , then retry Questions 3-6 and 9-12. At 14 or above: continue to the online module challenge.