

Arithmetic Sequences

Teacher guide and model answers

Chapter 2 | Module 6 | Student Book pages 40-56 | Suggested time: 50 minutes | 20 marks



Learning goals

- Explain and apply general term with valid conditions and clear reasoning.
- Explain and apply common difference with valid conditions and clear reasoning.
- Explain and apply finding parameters with valid conditions and clear reasoning.

Quick reference

General term	An arithmetic sequence has $a_n = a_1 + (n-1)d$.
Common difference	The common difference is $a_{n+1} - a_n$ and is constant.
Finding parameters	Two term conditions can determine the first term and common difference.

Worked example

An arithmetic sequence has $a_1=6$ and common difference 4. Find a_{11} . Worked solution:
 $a_{11} = a_1 + (11-1)d = 6 + 10 \cdot 4 = 46$.

Graduated practice

Answer without viewing the key. Show complete working for short-response questions.

1. An arithmetic sequence has first term 7 and common difference 5. Find a_{12} . [1 marks]

- A) 57
C) 62
- B) 60
D) 67

Answer: 62

$$a_{12} = 7 + (12-1) \times 5 = 62.$$

2. For the sequence 18, 13, 8, 3, ... what is the common difference? [1 marks]

- A) 5
C) -3
- B) -5
D) 31

Answer: -5

Subtract consecutive terms in the same order: $13-18=-5$ and $8-13=-5$.

3. In an arithmetic sequence, $a_4=17$ and $a_9=42$. Find a_1 and d . [1 marks]

- A) $a_1=2$, $d=5$
C) $a_1=7$, $d=5$
- B) $a_1=5$, $d=2$
D) $a_1=2$, $d=25$

Answer: $a_1=2$, $d=5$

$$5d = a_9 - a_4 = 25, \text{ so } d=5; \text{ then } a_1 = 17 - 3 \times 5 = 2.$$

4. An arithmetic sequence has first term 4 and common difference 3. Find the sum of its first 10 terms. [1 marks]

- A) 155
C) 175
- B) 165
D) 185

Answer: 175

$$S_{10} = 10/2 [2 \times 4 + (10-1) \times 3] = 5 \times 35 = 175.$$

5. Find the sum of all integers from 21 to 50 inclusive. [1 marks]

- A) 1035
C) 1065
- B) 1050
D) 1080

Answer: 1065

There are $50-21+1=30$ terms, so the sum is $30(21+50)/2=1065$.

6. If $a_n = 20 - 3(n-1)$, for which n is S_n greatest? [1 marks]

A) $n=6$

B) $n=7$

C) $n=8$

D) $n=20$

Answer: $n=7$

The terms stay positive through $a_7=2$, while $a_8=-1$; therefore the partial sums rise through S_7 and then fall.

7. True or false: the general term of an arithmetic sequence is $a_n = a_1 + nd$. [1 marks]

☐ True ☐ False

Answer: False

The correct formula is $a_n = a_1 + (n-1)d$ because there are $n-1$ steps from the first term to term n .

8. True or false: if the difference between each term and its predecessor is constant, the sequence is arithmetic. [1 marks]

☐ True ☐ False

Answer: True

A constant first difference is the defining property of an arithmetic sequence.

9. In an arithmetic sequence, $a_5=18$ and $a_{12}=46$. Find a_1 and d , showing your method. [3 marks]

Answer: $a_1=2$, $d=4$

$7d = 46 - 18 = 28$, so $d=4$; then $a_1 = 18 - 4 \times 4 = 2$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

10. Find the sum of the first 15 terms of 8, 12, 16, ... Write the formula and substitution. [3 marks]

Answer: 540

$a_{15} = 8 + 14 \times 4 = 64$, then $S_{15} = 15(8+64)/2 = 540$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

11. Find the sum of the multiples of 6 from 24 to 120 inclusive. Show how you counted the terms. [3 marks]

Answer: 1224

The number of terms is $(120-24)/6+1=17$, so the sum is $17(24+120)/2=1224$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

12. If $a_n = 32 - 5(n-1)$, find the greatest partial sum S_n and the value of n where it occurs. Justify your answer. [3 marks]

Answer: $S_7 = 119$ at $n = 7$

$a_7 = 2$ is positive and $a_8 = -3$ is negative, so the maximum is $S_7 = 7(32+2)/2 = 119$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

Challenge and review

1. Derive the sum of the first n terms of an arithmetic sequence by writing the sum forwards and backwards.

Write S_n forwards and backwards. Adding corresponding terms gives $2S_n = n(a_1 + a_n)$, hence $S_n = n(a_1 + a_n)/2$.

2. A student saves EGP 40 in week 1 and increases the amount by EGP 15 each week. Find the week-12 amount and the total saved over 12 weeks.

$a_{12} = 40 + 11 \times 15 = \text{EGP } 205$, and $S_{12} = 12(40 + 205)/2 = \text{EGP } 1,470$.

3. Error analysis: a student says there are $50 - 21 = 29$ integers from 21 to 50 inclusive. Explain and correct the error.

Both endpoints are included, so add 1: $50 - 21 + 1 = 30$ integers.

Follow-up plan

Below 14/20: revisit the difference between a_n and S_n and inclusive term counting, then retry Questions 3-6 and 9-12. At 14 or above: continue to the online module challenge.