Inequalities

Teacher guide and model answers

Module 5 | Student Book pages 31-39 | Suggested time: 50 minutes | 20 marks

Learning goals

- Explain and apply difference method with valid conditions and clear reasoning.
- Explain and apply completing the square with valid conditions and clear reasoning.
- Explain and apply equality condition with valid conditions and clear reasoning.

Quick reference

Difference method	To prove LHS>RHS, show that LHS-RHS is positive under the conditions.
Completing the square	A square is nonnegative for every real input and gives a universal bound.
Equality condition	A sharp inequality states when equality occurs.

Worked example

For t>0, find the minimum value of t + 25/t. Worked solution: AM-GM gives t+25/t ≥ 2√25=10, with equality at t=5.

Graduated practice

Answer without viewing the key. Show complete working for short-response questions.

1. For t>0, find the minimum value of t + 25/t. [1 marks]

A) 5 B) 25

C) 10 D) 12

Answer: 10

AM-GM gives $t+25/t \geq 2\sqrt{25}=10$, with equality at t=5.

2. For t>0, find the minimum value of t + 9/t. [1 marks]

A) 6 B) 3

C) 9 D) 8

Answer: 6

AM-GM gives $t+9/t \geq 2\sqrt{9}=6$, with equality at t=3.

3. For t>0, find the minimum value of t + 4/t. Use the “Equality condition” method. [1 marks]

A) 2 B) 6

C) 4 D) 4 + 1

Answer: 4

AM-GM gives $t+4/t \geq 2\sqrt{4}=4$, with equality at t=2.

4. For t>0, find the minimum value of t + 9/t. Use the “AM-GM” method. [1 marks]

A) 6 B) 3

C) 9 D) 8

Answer: 6

AM-GM gives $t+9/t \geq 2\sqrt{9}=6$, with equality at t=3.

5. For t>0, find the minimum value of t + 25/t. Use the “Positive-factor reasoning” method. [1 marks]

A) 5 B) 25

C) 10 D) 12

Answer: 10

AM-GM gives $t+25/t \geq 2\sqrt{25}=10$, with equality at t=5.

6. For t>0, find the minimum value of t + 16/t. Use the “Optimization bound” method. [1 marks]

A) 8 B) 4

C) 16 D) 10

Answer: 8

AM-GM gives t+16/t ≥ 2√16=8, with equality at t=4.

7. True or false: To prove LHS>RHS, show that LHS-RHS is positive under the conditions. [1 marks]

□ True □ False

Answer: True

True. To prove LHS>RHS, show that LHS-RHS is positive under the conditions.

8. True or false: A square can be negative when its inside is negative. [1 marks]

□ True □ False

Answer: False

False. A square is nonnegative for every real input and gives a universal bound.

9. For t>0, find the minimum value of t + 4/t. Show a complete method. [3 marks]

Answer: 4

AM-GM gives t+4/t ≥ 2√4=4, with equality at t=2.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

10. For t>0, find the minimum value of t + 64/t. Show a complete method. [3 marks]

Answer: 16

AM-GM gives t+64/t ≥ 2√64=16, with equality at t=8.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

11. For t>0, find the minimum value of t + 36/t. Show a complete method. [3 marks]

Answer: 12

AM-GM gives t+36/t ≥ 2√36=12, with equality at t=6.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

12. For t>0, find the minimum value of t + 36/t. Show a complete method using the “Optimization bound” method. [3 marks]

Answer: 12

AM-GM gives $t+36/t \geq 2\sqrt{36}=12$, with equality at $t=6$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

Challenge and review

1. Prove that for $t>0$, $t+16/t\geq 8$, and state the equality case.

Since t and $16/t$ are positive, AM-GM gives $t+16/t\geq 2\sqrt{16}=8$, with equality when $t=4$.

2. Create a minimum-value problem of the form $t+a/t$, choosing a so the minimum is an integer.

Accept any positive perfect-square a, for example $a=25$, giving a minimum of 10 at $t=5$.

3. Error analysis: a student applied AM-GM at $t=-2$. Explain why this is invalid.

This AM-GM form requires positive quantities; at $t=-2$ the positivity condition fails, so the method cannot be applied.

Follow-up plan

Below 14/20: revisit the positivity condition and equality case, then retry Questions 1-4 and 9. At 14 or above: continue to the online module challenge.