

Inequalities

Printable teaching and practice sheet

Module 5 | Student Book pages 31-39 | Suggested time: 50 minutes | 20 marks



Name: _____ Class: _____ Date: _____

Learning goals

- Explain and apply difference method with valid conditions and clear reasoning.
- Explain and apply completing the square with valid conditions and clear reasoning.
- Explain and apply equality condition with valid conditions and clear reasoning.

Quick reference

Difference method	To prove $LHS > RHS$, show that $LHS - RHS$ is positive under the conditions.
Completing the square	A square is nonnegative for every real input and gives a universal bound.
Equality condition	A sharp inequality states when equality occurs.

Worked example

For $t > 0$, find the minimum value of $t + 25/t$. Worked solution: AM-GM gives $t + 25/t \geq 2\sqrt{25} = 10$, with equality at $t = 5$.

Graduated practice

Answer without viewing the key. Show complete working for short-response questions.

1. For $t > 0$, find the minimum value of $t + 25/t$. [1 marks]

- | | |
|-------|-------|
| A) 5 | B) 25 |
| C) 10 | D) 12 |

2. For $t > 0$, find the minimum value of $t + 9/t$. [1 marks]

- | | |
|------|------|
| A) 6 | B) 3 |
| C) 9 | D) 8 |

3. For $t > 0$, find the minimum value of $t + 4/t$. Use the “Equality condition” method. [1 marks]

- | | |
|------|------------|
| A) 2 | B) 6 |
| C) 4 | D) $4 + 1$ |

4. For $t > 0$, find the minimum value of $t + 9/t$. Use the “AM-GM” method. [1 marks]

- | | |
|------|------|
| A) 6 | B) 3 |
| C) 9 | D) 8 |

5. For $t > 0$, find the minimum value of $t + 25/t$. Use the “Positive-factor reasoning” method. [1 marks]

- | | |
|-------|-------|
| A) 5 | B) 25 |
| C) 10 | D) 12 |

6. For $t > 0$, find the minimum value of $t + 16/t$. Use the “Optimization bound” method. [1 marks]

- | | |
|-------|-------|
| A) 8 | B) 4 |
| C) 16 | D) 10 |

7. True or false: To prove $LHS > RHS$, show that $LHS - RHS$ is positive under the conditions. [1 marks]

☐ True ☐ False

8. True or false: A square can be negative when its inside is negative. [1 marks]

☐ True ☐ False

9. For $t > 0$, find the minimum value of $t + 4/t$. Show a complete method. [3 marks]

10. For $t > 0$, find the minimum value of $t + 64/t$. Show a complete method. [3 marks]

11. For $t > 0$, find the minimum value of $t + 36/t$. Show a complete method. [3 marks]

12. For $t > 0$, find the minimum value of $t + 36/t$. Show a complete method using the “Optimization bound” method. [3 marks]

Challenge and review

1. Prove that for $t > 0$, $t + 16/t \geq 8$, and state the equality case.
2. Create a minimum-value problem of the form $t + a/t$, choosing a so the minimum is an integer.
3. Error analysis: a student applied AM-GM at $t = -2$. Explain why this is invalid.

Follow-up plan

Below 14/20: revisit the positivity condition and equality case, then retry Questions 1-4 and 9. At 14 or above: continue to the online module challenge.