

Identities and Proofs

Teacher guide and model answers

Module 4 | Student Book pages 21-30 | Suggested time: 50 minutes | 20 marks



Learning goals

- Explain and apply identity with valid conditions and clear reasoning.
- Explain and apply coefficient comparison with valid conditions and clear reasoning.
- Explain and apply one-sided proof with valid conditions and clear reasoning.

Quick reference

Identity	An identity is true for every value in its domain.
Coefficient comparison	Equal polynomials have equal coefficients for corresponding powers.
One-sided proof	A clean identity proof transforms one side independently into the other side.

Worked example

Use an identity to evaluate $(9+6)^2 - (9-6)^2$ without expanding both squares separately. Worked solution: Use $(u+v)^2 - (u-v)^2 = 4uv$. Thus $4 \cdot 9 \cdot 6 = 216$.

Graduated practice

Answer without viewing the key. Show complete working for short-response questions.

1. Use an identity to evaluate $(9+6)^2 - (9-6)^2$ without expanding both squares separately. [1 marks]

- A) 108
B) 60
C) 216
D) 220

Answer: 216

Use $(u+v)^2 - (u-v)^2 = 4uv$. Thus $4 \cdot 9 \cdot 6 = 216$.

2. Use an identity to evaluate $(9+3)^2 - (9-3)^2$ without expanding both squares separately. [1 marks]

- A) 108
B) 54
C) 48
D) 112

Answer: 108

Use $(u+v)^2 - (u-v)^2 = 4uv$. Thus $4 \cdot 9 \cdot 3 = 108$.

3. Use an identity to evaluate $(3+2)^2 - (3-2)^2$ without expanding both squares separately. [1 marks]

- A) 12
B) 20
C) 24
D) 28

Answer: 24

Use $(u+v)^2 - (u-v)^2 = 4uv$. Thus $4 \cdot 3 \cdot 2 = 24$.

4. Use an identity to evaluate $(6+4)^2 - (6-4)^2$ without expanding both squares separately. [1 marks]

- A) 96
B) 48
C) 40
D) 100

Answer: 96

Use $(u+v)^2 - (u-v)^2 = 4uv$. Thus $4 \cdot 6 \cdot 4 = 96$.

5. Use an identity to evaluate $(7+3)^2 - (7-3)^2$ without expanding both squares separately. [1 marks]

- A) 42
B) 40
C) 84
D) 88

Answer: 84

Use $(u+v)^2 - (u-v)^2 = 4uv$. Thus $4 \cdot 7 \cdot 3 = 84$.

6. Use an identity to evaluate $(4+6)^2 - (4-6)^2$ without expanding both squares separately. Use the “Proof logic” method. [1 marks]

A) 96

B) 48

C) 40

D) 100

Answer: 96

Use $(u+v)^2 - (u-v)^2 = 4uv$. Thus $4 \cdot 4 \cdot 6 = 96$.

7. True or false: An identity is true for every value in its domain. [1 marks]

☐ True ☐ False

Answer: True

True. An identity is true for every value in its domain.

8. True or false: Only constant terms need to match. [1 marks]

☐ True ☐ False

Answer: False

False. Equal polynomials have equal coefficients for corresponding powers.

9. Use an identity to evaluate $(4+4)^2 - (4-4)^2$ without expanding both squares separately. Show a complete method. [3 marks]

Answer: 64

Use $(u+v)^2 - (u-v)^2 = 4uv$. Thus $4 \cdot 4 \cdot 4 = 64$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

10. Use an identity to evaluate $(3+5)^2 - (3-5)^2$ without expanding both squares separately. Show a complete method. [3 marks]

Answer: 60

Use $(u+v)^2 - (u-v)^2 = 4uv$. Thus $4 \cdot 3 \cdot 5 = 60$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

11. Use an identity to evaluate $(5+6)^2 - (5-6)^2$ without expanding both squares separately. Show a complete method. [3 marks]

Answer: 120

Use $(u+v)^2 - (u-v)^2 = 4uv$. Thus $4 \cdot 5 \cdot 6 = 120$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

12. Use an identity to evaluate $(4+4)^2 - (4-4)^2$ without expanding both squares separately. Show a complete method using the “Proof logic” method. [3 marks]

Answer: 64

Use $(u+v)^2 - (u-v)^2 = 4uv$. Thus $4 \cdot 4 \cdot 4 = 64$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

Challenge and review

1. Prove $(a+b)^2-(a-b)^2=4ab$ by transforming one side only.

Start with the left side: $a^2+2ab+b^2-(a^2-2ab+b^2)=4ab$, which is the right side.

2. If $(px+q)(x+2)=3x^2+7x+2$ is an identity, find p and q by comparing coefficients.

Expanding gives $px^2+(2p+q)x+2q$. Hence $p=3$ and $q=1$; the middle coefficient check gives $2p+q=7$.

3. Error analysis: a student tested an identity only at $x=1$ and declared it proved. Explain the error.

One successful test value cannot prove a statement for every value in the domain; a valid algebraic transformation is required.

Follow-up plan

Below 14/20: revisit the difference between an identity and an equation, plus coefficient comparison, then retry Questions 1-4 and 9. At 14 or above: continue to the online module challenge.