Coordinate Geometry Basics

Teacher guide and model answers

Chapter 4 | Module 15 | Student Book pages 150-161 | Suggested time: 50 minutes | 20 marks

Learning goals

- Explain and apply directed division on a line with valid conditions and clear reasoning.
- Explain and apply distance between points with valid conditions and clear reasoning.
- Explain and apply section formula with valid conditions and clear reasoning.

Quick reference

Directed division on a line	Internal and external division use directed ratios to locate a point relative to two endpoints.
Distance between points	The distance is $\sqrt{[(x_{□}-x_{□})^2+(y_{□}-y_{□})^2]}$.
Section formula	A point dividing AB internally in ratio m:n is a weighted average of endpoint coordinates.

Worked example

For A(1,-2) and B(5,1), AB=√[(5-1)^2+(1-(-2))^2]=√25=5.

Graduated practice

Answer without viewing the key. Show complete working for short-response questions.

1. On a number line, P divides A=2 to B=10 internally with AP:PB=1:3. Find P. [1 marks]

A) 3
B) 4
C) 6
D) 8

Answer: 4

Use the weighted average $P=(3\times2+1\times10)/(1+3)=4$.

2. In which quadrant is (-3,4), and what is its reflection in the x-axis? [1 marks]

A) Quadrant I; (3,4)
B) Quadrant II; (-3,-4)
C) Quadrant III; (3,-4)
D) Quadrant IV; (-3,4)

Answer: Quadrant II; reflection (-3,-4)

Negative x and positive y place it in Quadrant II. Reflection in the x-axis changes only the sign of y.

3. Find the distance between A(1,-2) and B(5,1). [1 marks]

A) 3
B) 4
C) 5
D) 7

Answer: 5

$AB=\sqrt{[(5-1)^2+(1-(-2))^2]}=\sqrt{(16+9)}=5$.

4. What is the locus of points equidistant from A(-2,0) and B(2,0)? [1 marks]

A) The x-axis
B) The y-axis, x=0
C) The line y=2
D) The circle $x^2+y^2=4$

Answer: The y-axis, x=0

The locus is the perpendicular bisector of AB. Its midpoint is the origin and AB is horizontal, so the bisector is x=0.

5. P divides A(1,2)B(7,8) internally in the ratio AP:PB=2:1. Find P. [1 marks]

A) P(3,4)
B) P(4,5)
C) P(5,6)
D) P(6,7)

Answer: P(5,6)

$P=[1\times A+2\times B]/3$, so the coordinates are $((1+14)/3,(2+16)/3)=(5,6)$.

6. If M(1,2) is the midpoint of A(-3,5)B, find B. [1 marks]

A) B(-1,3)
B) B(4,-3)
C) B(5,-1)
D) B(2,-5)

Answer: B(5,-1)

From 2M=A+B, B=2M-A=(2,4)-(-3,5)=(5,-1).

7. True or false: reflection in the x-axis keeps the x-coordinate and reverses the sign of the y-coordinate. [1 marks]

□ True □ False

Answer: True

The transformation is (x,y) to (x,-y).

8. True or false: every point equidistant from A and B is the midpoint of AB. [1 marks]

□ True □ False

Answer: False

Every point on the perpendicular bisector is equidistant from A and B; the midpoint is only one such point.

9. On a number line A=2 and B=8. Find P that divides AB externally in the ratio AP:PB=3:1, and verify the ratio. [3 marks]

Answer: P=11

For external division P=(3×8-1×2)/(3-1)=11. Then AP=9 and PB=3, giving 3:1.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

10. If P(x,2) is equidistant from A(-1,0) and B(5,4), find x. [3 marks]

Answer: x=2

Equate squared distances: (x+1)^2+4=(x-5)^2+4. Thus (x+1)^2=(x-5)^2, giving x=2.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

11. P divides A(-2,1)B(8,6) internally in the ratio AP:PB=3:2. Find P. [3 marks]

Answer: P(4,4)

P=[2A+3B]/5, so x=(-4+24)/5=4 and y=(2+18)/5=4.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

12. B is the image of A(-3,4) under central symmetry about M(2,-1). Find B and check the midpoint. [3 marks]

Answer: B(7,-6)

B=2M-A=(4,-2)-(-3,4)=(7,-6). The midpoint of A and B is ((-3+7)/2,(4-6)/2)=(2,-1)=M.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

Challenge and review

1. Derive the coordinates of P dividing A(x1,y1)B(x2,y2) internally in the ratio AP:PB=m:n using vectors.

Since AP/AB=m/(m+n), P=A+[m/(m+n)](B-A). Hence P=[nA+mB]/(m+n), or ((nx1+mx2)/(m+n),(ny1+my2)/(m+n)).

2. Find the centre of the circle through A(0,0), B(6,0) and C(2,4) by using equal distances.

The perpendicular bisector of AB is x=3, so write O(3,y). From OA^2=OC^2, 9+y^2=1+(y-4)^2=17-8y+y^2, giving y=1; hence O(3,1).

3. Error analysis: a student uses denominator m+n in the external-section formula. Explain why m-n is required and state the condition m≠n.

For external division, the directed segments from P have opposite orientations, so the weights have opposite signs and the denominator is m-n. If m=n there is no finite external division point, hence m≠n.

Follow-up plan

Below 14/20: revisit reflection signs, squared distance, and internal and external section formulas, then retry Questions 2-6 and 9-12. At 14 or above: continue to the online module challenge.