

Higher-Degree Equations

Printable teaching and practice sheet

Chapter 3 | Module 14 | Student Book pages 142-149 | Suggested time: 50 minutes | 20 marks



Name: _____ Class: _____ Date: _____

Learning goals

- Explain and apply degree and number of roots with valid conditions and clear reasoning.
- Explain and apply cubic from roots with valid conditions and clear reasoning.
- Explain and apply repeated root with valid conditions and clear reasoning.

Quick reference

Degree and number of roots	A degree-n polynomial has n complex roots counted with multiplicity.
Cubic from roots	A cubic with roots r,s,t is proportional to $(x-r)(x-s)(x-t)$.
Repeated root	A repeated root is shared by $P(x)$ and its derivative $P'(x)$.

Worked example

To solve $x^3 - 4x^2 + x + 6 = 0$, verify $P(2) = 0$, then divide by $x - 2$ to get $(x - 2)(x - 3)(x + 1) = 0$; the roots are 2, 3 and -1.

Graduated practice

Answer without viewing the key. Show complete working for short-response questions.

1. How many roots does a degree-4 polynomial have over the complex numbers, counting multiplicity? [1 marks]

- A) 2
B) 3
C) 4
D) It cannot be determined

2. If 2 is a root of $x^3+kx^2-4x-4=0$, find k. [1 marks]

- A) -1
B) 0
C) 1
D) 2

3. Which monic cubic equation has roots -1, 2 and 3? [1 marks]

- A) $x^3-4x^2+x+6=0$
B) $x^3+4x^2-x-6=0$
C) $x^3-6x^2+x+4=0$
D) $x^3-4x^2-x-6=0$

4. If ω is a nonreal cube root of unity, what is ω^{2026} ? [1 marks]

- A) 1
B) ω
C) ω^2
D) -1

5. In $(x-2)^2(x+1)=0$, which root is repeated? [1 marks]

- A) -1 with multiplicity 2
B) 1 with multiplicity 2
C) 2 with multiplicity 2
D) 2 with multiplicity 3

6. What is the real solution set of $x^4-5x^2+4=0$? [1 marks]

- A) $\{-2, 2\}$
B) $\{-1, 1\}$
C) $\{-2, -1, 1, 2\}$
D) $\{1, 2\}$

7. True or false: every degree-n polynomial equation has n distinct real roots. [1 marks]

☐ True ☐ False

8. True or false: if r is a repeated root of P(x), then $P(r)=0$ and $P'(r)=0$. [1 marks]

☐ True ☐ False

9. Solve $x^3-4x^2+x+6=0$ using a known integer root and degree reduction. [3 marks]

10. Form the monic cubic equation whose roots are 1, -2 and 4. [3 marks]

11. If $\omega^3=1$ and $1+\omega+\omega^2=0$, simplify $(1+\omega)^6$. [3 marks]

12. If 1 is a repeated root of $P(x)=x^3+ax^2+bx+2$, find a and b, then factor $P(x)$. [3 marks]

Challenge and review

1. Derive Vieta's relations for a monic cubic with roots r , s and t by expanding $(x-r)(x-s)(x-t)$.
2. Solve $x^6-5x^3+4=0$ over the complex numbers, listing all six roots using ω .
3. Error analysis: when r is a root, a student writes the factor $x+r$. Correct the claim and state when $x+r$ would be the right factor.

Follow-up plan

Below 14/20: revisit root counting with multiplicity, degree reduction and cube roots of unity, then retry Questions 2-6 and 9-12. At 14 or above: continue to the online module challenge.