

Polynomial Theorems

Printable teaching and practice sheet

Chapter 3 | Module 13 | Student Book pages 132-141 | Suggested time: 50 minutes | 20 marks



Name: _____ Class: _____ Date: _____

Learning goals

- Explain and apply remainder theorem with valid conditions and clear reasoning.
- Explain and apply unknown coefficients with valid conditions and clear reasoning.
- Explain and apply factor theorem with valid conditions and clear reasoning.

Quick reference

Remainder theorem	The remainder when $P(x)$ is divided by $x-a$ is $P(a)$.
Unknown coefficients	Remainder conditions become equations in unknown polynomial coefficients.
Factor theorem	$x-a$ is a factor of $P(x)$ exactly when $P(a)=0$.

Worked example

For the remainder when $P(x)=2x^3-3x+4$ is divided by $x-2$, substitute $x=2$ directly: $P(2)=16-6+4=14$.

Graduated practice

Answer without viewing the key. Show complete working for short-response questions.

1. Find the remainder when $P(x)=2x^3-3x+4$ is divided by $x-2$. [1 marks]

- A) 6
B) 10
C) 14
D) 18

2. Find the remainder when $P(x)=4x^2+2x-1$ is divided by $2x+3$. [1 marks]

- A) -5
B) 1
C) 5
D) 11

3. The remainder when $P(x)=x^3+kx+2$ is divided by $x-1$ is 7. Find k . [1 marks]

- A) 2
B) 3
C) 4
D) 6

4. What is the quotient when x^3-4x^2+x+6 is divided by $x-2$? [1 marks]

- A) x^2-2x-3
B) x^2-6x-3
C) x^2+2x-3
D) x^2-2x+3

5. Which expression is a factor of $P(x)=x^3-4x$? [1 marks]

- A) $x-1$
B) $x-2$
C) $x-3$
D) $x-4$

6. What is the complete factorisation of $x^3-6x^2+11x-6$? [1 marks]

- A) $(x+1)(x+2)(x+3)$
B) $(x-1)(x-2)(x-3)$
C) $(x-1)(x+2)(x-3)$
D) $(x+1)(x-2)(x+3)$

7. True or false: when using synthetic division on x^3+5 by $x-2$, the coefficients of x^2 and x must be entered as zeros. [1 marks]

☐ True ☐ False

8. True or false: if $P(a)=0$, then $x+a$ is a factor of $P(x)$. [1 marks]

☐ True ☐ False

9. If $x-1$ and $x+2$ are factors of $P(x)=x^3+ax^2+bx+6$, find a and b . [3 marks]

10. Divide $2x^3+3x^2-8x-12$ by $x+2$ using synthetic division. State the quotient and remainder. [3 marks]

11. Solve $2x^3-3x^2-8x+12=0$ using the Factor Theorem and degree reduction. [3 marks]

12. When $P(x)$ is divided by $(x-1)(x+2)$, $P(1)=3$ and $P(-2)=-6$. Find the remainder. [3 marks]

Challenge and review

1. Prove that $P(x)-P(a)$ is divisible by $x-a$ for every polynomial P , then deduce the Remainder Theorem.

2. Factor x^4-5x^2+4 completely using the Factor Theorem and degree reduction.

3. Error analysis: to find a remainder on division by $2x-1$, a student substitutes $x=2$. Identify the correct substitution and explain where it comes from.

Follow-up plan

Below 14/20: revisit the divisor root, zero coefficients, and the Remainder and Factor Theorems, then retry Questions 2-6 and 9-12. At 14 or above: continue to the online module challenge.