Complex Numbers

Teacher guide and model answers

Chapter 3 | Module 11 | Student Book pages 106-114 | Suggested time: 50 minutes | 20 marks

Learning goals

- Explain and apply real and imaginary parts with valid conditions and clear reasoning.
- Explain and apply powers of i with valid conditions and clear reasoning.
- Explain and apply conjugate and division with valid conditions and clear reasoning.

Quick reference

Real and imaginary parts	A complex number has standard form $a+bi$ with real part a and imaginary part b.
Powers of i	Powers of i repeat every four: $1, i, -1, -i$.
Conjugate and division	The conjugate $a-bi$ makes $(a+bi)(a-bi)=a^2+b^2$ real.

Worked example

If $z=-3+5i$, then $\mathrm{Re}(z)=-3$ and $\mathrm{Im}(z)=5$; the imaginary part is the real coefficient of i.

Graduated practice

Answer without viewing the key. Show complete working for short-response questions.

1. If z=-3+5i, what are its real and imaginary parts? [1 marks]

A) Re(z)=-3, Im(z)=5

B) Re(z)=5, Im(z)=-3

C) Re(z)=-3, Im(z)=5i

D) Re(z)=3, Im(z)=5

Answer: Re(z)=-3, Im(z)=5

In a+bi, the real part is a and the imaginary part is the real coefficient b.

2. If (2x-1)+(y+3)i=7-2i, find x and y. [1 marks]

A) x=3, y=-5

B) x=4, y=-5

C) x=4, y=1

D) x=-4, y=5

Answer: x=4, y=-5

Equate parts: 2x-1=7 gives x=4, while y+3=-2 gives y=-5.

3. What is i^2026? [1 marks]

A) 1

B) i

C) -1

D) -i

Answer: -1

Powers of i repeat every 4. Since 2026 leaves remainder 2 on division by 4, i^2026=i^2=-1.

4. Calculate (3-2i)(1+4i) and write the result as a+bi. [1 marks]

A) -5+10i

B) 11+10i

C) 11-10i

D) 3+8i

Answer: 11+10i

Expand: 3+12i-2i-8i^2=3+10i+8=11+10i.

5. Simplify (4+2i)/(1-i) to the form a+bi. [1 marks]

A) 1+3i

B) 3+i

C) 1-3i

D) 2+6i

Answer: 1+3i

Multiply by the conjugate 1+i. The numerator becomes 2+6i and the denominator 2, giving 1+3i.

6. Solve $x^2+49=0$ in the complex numbers. [1 marks]

A) $x=7i$ only

B) $x=-7i$ only

C) $x=\pm 7$

D) $x=\pm 7i$

Answer: $x=\pm 7i$

$x^2=-49$, so $x=\pm\sqrt{-49}=\pm 7i$.

7. True or false: if $z=a+bi$, then z times its conjugate is a^2+b^2, a non-negative real number. [1 marks]

□ True □ False

Answer: True

$(a+bi)(a-bi)=a^2-(bi)^2=a^2+b^2$.

8. True or false: $\sqrt{-4}\times\sqrt{-9}=\sqrt{36}=6$. [1 marks]

□ True □ False

Answer: False

Using principal roots, $\sqrt{-4}=2i$ and $\sqrt{-9}=3i$, so the product is $6i^2=-6$, not 6.

9. Simplify $i^{37}+i^{58}+i^{99}$, showing how the four-term cycle is used. [3 marks]

Answer: -1

$i^{37}=i$, $i^{58}=-1$ and $i^{99}=-i$, so the sum is $i-1-i=-1$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

10. If $(a+2)+(3b-1)i=5+8i$, find a and b and justify the comparison of parts. [3 marks]

Answer: $a=3$, $b=3$

Equating real parts gives $a+2=5$, so $a=3$. Equating imaginary parts gives $3b-1=8$, so $b=3$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

11. Simplify $(2+3i)/(2-3i)$ to $a+bi$, showing the use of the conjugate. [3 marks]

Answer: $(-5+12i)/13$

Multiply numerator and denominator by $2+3i$. The numerator is $(2+3i)^2=-5+12i$ and the denominator is $2^2+3^2=13$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

12. Solve $x^2-6x+13=0$ in the complex numbers and check the two roots. [3 marks]

Answer: $x=3\pm2i$

The discriminant is $36-52=-16$, so $x=(6\pm4i)/2=3\pm2i$. Their sum is 6 and product is 13.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

Challenge and review

1. Prove that if $z=a+bi$ is nonzero, then $1/z=(a-bi)/(a^2+b^2)$.

> Multiply numerator and denominator by the conjugate: $1/(a+bi)=(a-bi)/[(a+bi)(a-bi)]=(a-bi)/(a^2+b^2)$, whose denominator is nonzero because z is nonzero.

2. If z and its conjugate satisfy z+conjugate(z)=8 and z-conjugate(z)=-6i, find z.

> Adding the equations gives $2z=8-6i$, so $z=4-3i$.

3. Error analysis: a student writes $\sqrt{-a}\sqrt{-b}=\sqrt{ab}$ for positive a and b. Explain the error and give the correct product of principal roots.

> Since $\sqrt{-a}=i\sqrt{a}$ and $\sqrt{-b}=i\sqrt{b}$, their product is $i^2\sqrt{ab}=-\sqrt{ab}$. The usual radical product rule cannot be transferred without its domain conditions.

Follow-up plan

Below 14/20: revisit the cycle of i, equality of parts and use of the conjugate, then retry Questions 2-6 and 9-12. At 14 or above: continue to the online module challenge.