Recurrence and Induction

Printable teaching and practice sheet

Chapter 2 | Module 10 | Student Book pages 91-105 | Suggested time: 50 minutes | 20 marks

Name: ____________________ Class: ____________________ Date: ____________________

Learning goals

• Explain and apply recurrence evaluation with valid conditions and clear reasoning.

• Explain and apply linear recurrence transformation with valid conditions and clear reasoning.

• Explain and apply induction step with valid conditions and clear reasoning.

Quick reference

Recurrence evaluation	A recurrence defines later terms from earlier terms plus enough initial data.
Linear recurrence transformation	A recurrence a_{n+1}=pa_n+c can be shifted by a fixed point to become geometric.
Induction step	The induction step proves that truth at k forces truth at k+1.

Worked example

Given u1=3 and u_(n+1)=u_n+4, find u12. Worked solution: The recurrence adds 4 exactly 11 times: u12=3+11·4=47.

Graduated practice

Answer without viewing the key. Show complete working for short-response questions.

1. If $u_1=2$ and $u_{(n+1)}=3u_n-1$, find u_4. [1 marks]

A) 14
B) 29
C) 41
D) 53

2. If $a_1=5$ and $a_{(n+1)}=a_n+4$, what is the explicit form of a_n? [1 marks]

A) $a_n=4n+1$
B) $a_n=5n+4$
C) $a_n=4n+5$
D) $a_n=5+4n$

3. If $a_1=1$ and $a_{(n+1)}=2a_n+3$, what is a_4? [1 marks]

A) 21
B) 27
C) 29
D) 32

4. To prove $1+3+5+...+(2n-1)=n^2$ by induction from $n=1$, which is the correct base case? [1 marks]

A) $1=1^2$
B) $1+3=2^2$
C) $2n-1=n^2$
D) Assume the result at $n=1$

5. In proving $1+2+...+n=n(n+1)/2$, assume $P(k)$. What expression results after adding $k+1$? [1 marks]

A) $k(k+1)/2$
B) $(k+1)^2/2$
C) $(k+1)(k+2)/2$
D) $k(k+2)/2$

6. To prove that 5^n-1 is divisible by 4, which rearrangement supports the induction step? [1 marks]

A) $5^{(k+1)}-1=5(5^k-1)+4$
B) $5^{(k+1)}-1=5^k+4$
C) $5^{(k+1)}-1=4(5^k-1)$
D) $5^{(k+1)}-1=5(5^k+1)$

7. True or false: a valid base case and proof that $P(k)$ implies $P(k+1)$ establish $P(n)$ for every integer from the starting value onward. [1 marks]

□ True □ False

8. True or false: checking the first 100 cases of a statement is an induction proof for all natural numbers. [1 marks]

□ True □ False

9. If $u_1=1$, $u_2=3$ and $u_{(n+2)}=u_{(n+1)}+2u_n$, find u_6 and show the intermediate terms. [3 marks]

10. If $a1=3$ and $a\_(n+1)=a\_n+2n$, derive an explicit form for a_n and then find $a8$. [3 marks]

11. Prove by induction that the sum of the first n odd numbers is n^2 for every $n\geq1$. [3 marks]

12. Prove by induction that 7^n-1 is divisible by 6 for every $n\geq1$. [3 marks]

Challenge and review

1. Solve $a1=1$, $a_{(n+1)}=2a_n+3$ by a suitable shift and write an explicit formula for a_n.

2. Prove by induction that $2^n \geq n+1$ for every $n \geq 0$.

3. Error analysis: in the induction step, a student assumes P(k+1) and rearranges it until reaching P(k+1). Explain why the proof is circular.

Follow-up plan

Below 14/20: revisit initial data, fixed-point shifts, and the base and induction steps, then retry Questions 3-6 and 9-12. At 14 or above: continue to the online module challenge.