

Binomial and Multinomial Theorems

Teacher guide and model answers

Module 1 | Student Book pages 4-9 | Suggested time: 50 minutes | 20 marks



Learning goals

- Explain and apply binomial coefficient with valid conditions and clear reasoning.
- Explain and apply general binomial term with valid conditions and clear reasoning.
- Explain and apply multinomial coefficient with valid conditions and clear reasoning.

Quick reference

Binomial coefficient	The number $C(n,r)$ counts selections and gives the coefficient pattern in a binomial expansion.
General binomial term	A term in $(a+b)^n$ has the form $C(n,r)a^{n-r}b^r$.
Multinomial coefficient	For powers p,q,r with $p+q+r=n$, the coefficient is $n!/(p!q!r!)$.

Worked example

Find the coefficient of x^3 in $(x + 4)^4$. Worked solution: The selected term is $C(4,3) \cdot x^3 \cdot 4^1$; therefore the coefficient is $C(4,3) \cdot 4^1 = 16$.

Graduated practice

Answer without viewing the key. Show complete working for short-response questions.

1. Find the coefficient of x^3 in $(x + 4)^4$. [1 marks]

- A) 256
B) 20
C) 16
D) 12

Answer: 16

The selected term is $C(4,3) \cdot x^3 \cdot 4^1$; therefore the coefficient is $C(4,3) \cdot 4^1 = 16$.

2. Find the coefficient of x^2 in $(x + 4)^5$. [1 marks]

- A) 640
B) 160
C) 644
D) 636

Answer: 640

The selected term is $C(5,2) \cdot x^2 \cdot 4^3$; therefore the coefficient is $C(5,2) \cdot 4^3 = 640$.

3. Find the coefficient of x^1 in $(x + 2)^6$. [1 marks]

- A) 12
B) 194
C) 192
D) 190

Answer: 192

The selected term is $C(6,1) \cdot x^1 \cdot 2^5$; therefore the coefficient is $C(6,1) \cdot 2^5 = 192$.

4. Find the coefficient of x^3 in $(x + 2)^5$. [1 marks]

- A) 40
B) 80
C) 42
D) 38

Answer: 40

The selected term is $C(5,3) \cdot x^3 \cdot 2^2$; therefore the coefficient is $C(5,3) \cdot 2^2 = 40$.

5. Find the coefficient of x^1 in $(x + 2)^8$. [1 marks]

- A) 16
B) 1026
C) 1024
D) 1022

Answer: 1024

The selected term is $C(8,1) \cdot x^1 \cdot 2^7$; therefore the coefficient is $C(8,1) \cdot 2^7 = 1024$.

6. Find the coefficient of x^1 in $(x + 3)^8$. [1 marks]

A) 17496

B) 24

C) 17499

D) 17493

Answer: 17496

The selected term is $C(8,1) \cdot x^1 \cdot 3^7$; therefore the coefficient is $C(8,1) \cdot 3^7 = 17496$.

7. True or false: The number $C(n,r)$ counts selections and gives the coefficient pattern in a binomial expansion. [1 marks]

☐ True ☐ False

Answer: True

True. The number $C(n,r)$ counts selections and gives the coefficient pattern in a binomial expansion.

8. True or false: Both exponents in a selected term are equal to r . [1 marks]

☐ True ☐ False

Answer: False

False. A term in $(a+b)^n$ has the form $C(n,r)a^{(n-r)}b^r$.

9. Find the coefficient of x^3 in $(x + 2)^7$. Show a complete method. [3 marks]

Answer: 560

The selected term is $C(7,3) \cdot x^3 \cdot 2^4$; therefore the coefficient is $C(7,3) \cdot 2^4 = 560$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

10. Find the coefficient of x^3 in $(x + 4)^6$. Show a complete method. [3 marks]

Answer: 1280

The selected term is $C(6,3) \cdot x^3 \cdot 4^3$; therefore the coefficient is $C(6,3) \cdot 4^3 = 1280$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

11. Find the coefficient of x^3 in $(x + 5)^4$. Show a complete method. [3 marks]

Answer: 20

The selected term is $C(4,3) \cdot x^3 \cdot 5^1$; therefore the coefficient is $C(4,3) \cdot 5^1 = 20$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

12. Find the coefficient of x^2 in $(x + 5)^5$. Show a complete method. [3 marks]

Answer: 1250

The selected term is $C(5,2) \cdot x^2 \cdot 5^3$; therefore the coefficient is $C(5,2) \cdot 5^3 = 1250$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

Challenge and review

1. Write a general rule for the coefficient of x^k in $(x+a)^n$ and define every symbol.

The coefficient is $C(n,k)a^{n-k}$, with $0 \leq k \leq n$.

2. Create your own sum-of-coefficients problem and solve it in two ways.

Accept a valid problem using $P(1)$, checked by expansion or direct calculation.

3. Error analysis: a student wrote $C(7,2)=7^2$. Explain the error and give the correct value.

$C(7,2)=7!/2!5!=21$; it counts selections, not a power.

Follow-up plan

Below 14/20: revisit the general term and binomial coefficient, then retry Questions 1-4 and 9. At 14 or above: continue to the online module challenge.