# Mathematics Book One - Chapter Two Assessment

Marking guide and explained answers

Modules 6-10 | 20 questions | 30 marks | 60 minutes

## Examination instructions

• Answer every question and show complete working for written responses.

• Do not view the marking guide until the examination is complete.

• For induction, state the base case, induction hypothesis and induction step clearly.

## Curriculum and mark balance

| | | | |
|---|---|---|---|
| 6 | Arithmetic sequences | 4 | 6 |
| 7 | Geometric sequences | 4 | 6 |
| 8 | Summation | 4 | 6 |
| 9 | Advanced sequences | 4 | 6 |
| 10 | Recurrence and induction | 4 | 6 |

# Section A: Objective questions - 15 marks

Choose the correct answer, then complete the true-or-false items.

**1. An arithmetic sequence has first term 7 and common difference 5. Find a12. [1 marks]**

A) 57
B) 60
C) 62
D) 67

**Answer: 62**

$a_{12}=7+(12-1)\times5=62$.

**2. An arithmetic sequence has first term 4 and common difference 3. Find the sum of its first 10 terms. [1 marks]**

A) 155
B) 165
C) 175
D) 185

**Answer: 175**

$S_{10}=10/2[2\times4+(10-1)\times3]=5\times35=175$.

**3. For the geometric sequence 162, 54, 18, 6, ... what is the common ratio? [1 marks]**

A) 3
B) 1/3
C) -3
D) -108

**Answer: 1/3**

Divide a term by its predecessor: $54/162=18/54=1/3$.

**4. A geometric sequence has first term 3 and common ratio 2. Find the sum of its first 6 terms. [1 marks]**

A) 96
B) 126
C) 189
D) 192

**Answer: 189**

$S_6=3(2^6-1)/(2-1)=3\times63=189$.

**5. Which expanded expression equals Σ from k=2 to 5 of (2k-1)? [1 marks]**

A) 1+3+5+7
B) 3+5+7+9
C) 3+5+7
D) 2+4+6+8

**Answer: 3+5+7+9**

Substitute $k=2$, 3, 4 and 5 into $2k-1$.

**6. Evaluate Σ from k=1 to 8 of 1/[k(k+1)]. [1 marks]**

A) 1/9
B) 7/8
C) 8/9
D) 9/8

**Answer: 8/9**

Since 1/[k(k+1)]=1/k-1/(k+1), intermediate terms cancel and 1-1/9=8/9 remains.

**7. If u_n=2n^2-3n, what is u_(n+1)-u_n? [1 marks]**

A) 2n-3
B) 4n-1
C) 4n+1
D) 2n^2-1

**Answer: 4n-1**

Substitute and subtract: 2(n+1)^2-3(n+1)-(2n^2-3n)=4n-1.

**8. If group sizes are 1,2,3,..., in which group and local position is term 23? [1 marks]**

A) Group 6, position 2
B) Group 7, position 2
C) Group 7, position 3
D) Group 8, position 1

**Answer: Group 7, position 2**

There are 21 terms through group 6; therefore term 23 is in group 7 at local position 23-21=2.

**9. If u1=2 and u_(n+1)=3u_n-1, find u4. [1 marks]**

A) 14
B) 29
C) 41
D) 53

**Answer: 41**

u2=5, then u3=14, then u4=41.

**10. To prove 1+3+5+...+(2n-1)=n^2 by induction from n=1, which is the correct base case? [1 marks]**

A) 1=1^2
B) 1+3=2^2
C) 2n-1=n^2
D) Assume the result at n=1

**Answer: 1=1^2**

Substitute n=1 into both sides: the left side is 1 and the right side is 1^2=1.

**11. True or false: if the difference between each term and its predecessor is constant, the sequence is arithmetic. [1 marks]**

□ True □ False

**Answer: True**

A constant first difference is the defining property of an arithmetic sequence.

**12. True or false: if the common ratio is negative, every term of the sequence is negative. [1 marks]**

□ True □ False

**Answer: False**

A negative ratio makes the signs alternate; it does not make every term negative.

**13. True or false: Σ from k=1 to n of the constant c equals c. [1 marks]**

□ True □ False

**Answer: False**

The constant c occurs n times, so the sum is nc.

**14. True or false: knowing only the difference sequence uniquely determines the original sequence. [1 marks]**

□ True □ False

**Answer: False**

An initial term is also required; sequences with the same differences may differ by a constant.

**15. True or false: checking the first 100 cases of a statement is an induction proof for all natural numbers. [1 marks]**

□ True □ False

**Answer: False**

A finite check cannot establish infinitely many cases; a valid base case and induction step are required.

# Section B: Written solutions - 15 marks

Show a complete, justified method. Each question is worth 3 marks.

**16. Find the sum of the multiples of 6 from 24 to 120 inclusive. Show how you counted the terms. [3 marks]**

**Answer: 1224**

The number of terms is $(120-24)/6+1=17$, so the sum is $17(24+120)/2=1224$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

**17. A geometric sequence has first term 64 and ratio 1/2. Find a7 and S7, and explain the effect of |r|<1. [3 marks]**

**Answer: a7=1, S7=127**

$a_7=64(1/2)^6=1$ and $S_7=64[1-(1/2)^7]/(1-1/2)=127$; term magnitudes decrease.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

**18. Evaluate Σ from k=1 to 5 of k×2^(k-1), displaying the terms before adding. [3 marks]**

**Answer: 129**

The sum is $1+4+12+32+80=129$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

**19. For 3,8,15,24,... construct a quadratic general term, then use it to find a6. [3 marks]**

**Answer: a_n=n^2+2n and a6=48**

The differences 5,7,9 have constant second difference 2. The formula $n^2+2n$ reproduces the data and gives $a_6=36+12=48$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

**20. Prove by induction that the sum of the first n odd numbers is n^2 for every n≥1. [3 marks]**

**Answer: The statement is true for every n≥1**

Base $n=1$: $1=1^2$. If the sum through $2k-1$ is $k^2$, adding $2k+1$ gives $k^2+2k+1=(k+1)^2$.

Rubric: valid method 1 | correct substitution/calculation 1 | clear checked result 1

# Mark summary and follow-up

| | | |
|---|---|---|
| **27-30** | Mastery | Begin Chapter Three with an extension challenge. |
| **23-26** | Secure | Review and retry two missed questions. |
| **18-22** | Developing | Redo the weakest module sheets before retesting. |
| **0-17** | Needs support | Return to instruction and worked examples with a teacher. |

Strongest module: ________________________________________________

Module to revisit: ________________________________________________

Retest date: ________________________________________________